A top-down view of the classical limit of quantum mechanics

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1.- Introduction

Since its birth in the early twentieth century, quantum mechanics raised a number of questions and problems, many of which are still a source of lively debate. The attempts to address those issues have led to a multiplicity of interpretations and theoretical developments which have enriched the scientific knowledge about the theory. Perhaps the problem most discussed in this context is the so-called *quantum measurement problem*, based on the theoretical difficulty to explain how measuring devices with classical pointers are able to produce results when acting on quantum systems (von Neumann 1932, Ballentine 1990, Bub 1997). Another question that has been the subject of intensive research is the problem of *the classical limit of quantum mechanics* (Bohm 1951, Schlosshauer 2007). According the correspondence principle (Bohr 1920; for a recent discussion, see Bokulich 2014), there should be a limiting procedure that accounts for the classical behavior of a system in terms of the laws of quantum mechanics. The problem of classical limit consists in explaining how the classical realm "emerges" from the quantum domain. The two problems just mentioned have something in common: both point to the need for finding a link between the classical and the quantum world.

Along the history of quantum mechanics, the classical limit has been approached from many different perspectives, such as those given by the Ehrenfest theorem (Ehrenfest 1927), the Wigner transform (Wigner 1932) and the deformation theory (Bayen *et al.* 1977, 1978). Traditionally, the problem was conceived as a matter of intertheory relation: classical mechanics should be obtained from quantum mechanics by means of the application of a mathematical limit, in a way analogous to the way in which the classical equations of motion are obtained from special relativity. However, this approach has been weakening over the past decades: at present it is recognized that the classical limit also involves some kind of physical process, which transforms the quantum states in such a way that they finally can be interpreted as classical states. This process is now known as *quantum decoherence*.

One the main features of quantum mechanics is the superposition principle, which leads to the phenomenon of quantum interference, without classical analogue. Decoherence is viewed as a process that cancels interference and selects the candidates to classical states. The cancellation of interference has been traditionally conceived in terms of the transformation of a pure state into a mixture without interference terms. From a geometrical viewpoint, the state of a quantum system passes from the frontier of the convex set of states (pure states) to its interior (mixture states) (Bengtsson and Yczkowski 2006; for a recent approach see Holik and Plastino 2011). On this basis, decoherence was studied in open and closed systems. Schematically, three periods can be identified in the historical development in the general program of decoherence (see Castagnino *et al*. 2008):

- **First period**: Several authors (van Kampen 1954, van Hove 1957, 1959, Daneri, Loinger and Prosperi 1962) studied the approach to equilibrium through the behavior of the so-called *collective observables*, that is, the observables accessible from the macroscopic viewpoint. This approach was based on the traditional methods used to describe irreversible processes. The aim was to understand how the classical macroscopic properties emerge from the quantum microscopic evolution. For this purpose, a coarse grained state $\rho_G(t)$ is defined, which carries all the macroscopic information of the system, and it is shown that, under certain definite conditions, $\rho_G(t)$ decoheres in the eigenbasis of $\rho_G(t)$ in a decoherence time t_D and reaches equilibrium after a relaxation time t_R. The main problem of this period was the fact that the decoherence time t_D computed with these primitive formalisms proved to bee too long when compared with experimental results (Omnès 2005).

- **Second period**: The interest is focused on open systems. An open quantum system S is considered in interaction with its environment E, and the time evolution of the reduced state $\rho_S(t) = Tr_E(\rho(t))$ is studied. According to the *environment-induced decoherence* (EID) approach (see, for instance, Zeh 1970, 1971, 1973, Zurek 1982, 1993, 2003), decoherence is the result of the interaction between the system S and the environment E. It is shown that, under certain definite conditions, the states of E become orthogonal in a very short decoherence time t_D and, as a consequence, interference disappears from the state $\rho_S(t)$ of S: it is said that $\rho_S(t)$ decoheres in a preferred basis. This solves the problem of the first period. Moreover, the formalism counts with many successful applications (see Joos *et al*. 2003).

  Under the assumption that quantum systems are never isolated and interact significantly with their environment (Zeh 1970), the EID approach was initially conceived to study the measurement problem, but was immediately extrapolated to the case of the classical limit. Nevertheless, it was questioned regarding its capacity of accounting of the emergence of classicality due to some conceptual difficulties (that will be discussed below).

- **Third period**: Although the EID approach is still the most widespread perspective, other perspectives have been proposed to deal with cases that are beyond the applicability domain of EID, in particular, the case of closed systems (Diosi 1987, 1989, Milburn 1991, Penrose 1995, Casati and Chirikov 1995a, 1995b, Adler 2003). Some of them were designed specifically to describe processes that do not dissipate energy to the environment (see Polarski and Starobinsky 1996, Bonifacio *et al*. 2000, Ford and O'Connell 2001, Frasca 2003, Sicardi *et al*. 2003, Gambini, Porto and Pullin 2007, Gambini and Pullin 2007, 2010, Kiefer and Polarski 2009). A formalism specifically devoted to describe decoherence in closed systems is the self-induced decoherence approach, according to which a closed system with continuous spectrum may decohere due to destructive interference (Castagnino 1999, 2004, 2006, Castagnino and Lombardi 2003, 2004, 2005a, 2005b, Castagnino and Ordóñez 2004, Castagnino and Gadella 2006, Castagnino and Fortin 2011a, 2011b, 2012, 2013).

The classical limit based on the EID approach describes a classical world that emerges from the interaction of the subsystems of a quantum system: this approach is a *bottom-up view*, since it begins by the analysis of the subsystems of the whole system and their interactions. In this article we will propose an inverse perspective, a *top-down view*, which begins by analyzing the whole closed system and its evolution, and on this basis identifies the degrees of freedom that will behave classically. From this perspective, decoherence is not a yes-or-no process, but a phenomenon *relative* to the decomposition of the closed system selected in each case. In turn, the classical limit based on this top-down view is more general than that resulting from the traditional EID approach, and dissolves some of its conceptual difficulties.

In order to develop our presentation, the article is organized as follows. In Section 2, the conceptual basis of the EID approach will be recalled, and in Section 3 its conceptual difficulties will be pointed out. Section 4 will be devoted to argue that reduced states are a kind of coarse-grained states of closed composite systems, and that it is for this reason that they cancel the correlations of a subsystem with other subsystems with which it interacts. In Section 5, the closed-system approach to decoherence will be introduced by showing that the loss of coherence in a system can be studied by considering the internal structure of the expectation values of its observables. On this basis, Section 6 will show that the EID approach can be reformulated from the open-system perspective, and Section 7 will explain how its conceptual difficulties are solved or dissolved from the new approach to the decoherence. Finally, Section 8 will be devoted to consider how the closed-system approach fits into a general top-down view of quantum mechanics, which underlies certain conceptual stances regarding the interpretation of the theory.

2.- Environment-induced decoherence: an open-system approach

Let us consider an open system S represented by a Hilbert space $\mathcal{H}_S$, whose the initial state $\rho_S(0)$ belongs to the Liouville space $\mathcal{L}_S = \mathcal{H}_S \otimes \mathcal{H}_S$, and that is in interaction with an environment E represented by a Hilbert space $\mathcal{H}_E$, whose the initial state $\rho_E(0)$ belongs to the Liouville space $\mathcal{L}_E = \mathcal{H}_E \otimes \mathcal{H}_E$. Then, the initial state $\rho_U(0)$ whole composite system $U = S \cup E$ is obtained as $\rho_U(0) = \rho_S(0) \otimes \rho_E(0)$. Therefore, at the initial time $t = 0$ it is possible to recover the initial states of S and E from the initial state of the closed system U by means of partial trace:

$$\rho_S(0) = Tr_E\left(\rho_U(0)\right) \qquad \rho_E(0) = Tr_S\left(\rho_U(0)\right) \tag{1}$$

The evolution of the total system is governed by the Hamiltonian $H_U = H_S + H_E + H_{SE}$, where H_S is the self-Hamiltonian of the system S, H_E is the self-Hamiltonian of the environment E, and H_{SE} is the interaction Hamiltonian. With H_U, the time evolution of the closed system U, represented by $\rho_U(t)$, can be computed by means of the Liouville-von Neumann equation.

Since partial trace recovers the initial state of S from the initial state of U, from the open-system perspective it is usual to adopt an additional hypothesis: S preserves its identity as a quantum system during the time evolution, and its state $\rho_S(t)$ can be computed in a manner analogous to that used in the initial time (eq.(1)):

$$\rho_S(t) = Tr_E\left(\rho_U(t)\right) \tag{2}$$

The EID formalism proves that, in many physical relevant models, the non-diagonal terms of the reduced state $\rho_S(t)$ rapidly tend to vanish after an extremely short decoherence time t_D:

$$\rho_S(t) \longrightarrow \rho_S^d(t) \tag{3}$$

where $\rho_S^d(t)$ is diagonal in the pointer basis. In fact, since the state $\rho_S(t)$ is a Hermitian operator, then it can always be diagonalized in the Schmidt basis. In this way, it is said that the system S decoheres as a consequence of its interaction with the large number of degrees of freedom of the environment E.

It is important to recall that, according to EID, decoherence amounts not only to the diagonalization of the reduced state, but to the diagonalization of the reduced state in the “preferred basis” or “pointer basis”, that is, the basis that defines what observables behave classically. Although the precise definition and the strategies to compute this preferred basis have been the subject of much discussion and criticism, the treatment of this point is beyond the limits of the present article (for details see Zurek 1993, 2009, Zurek, Habib and Paz 1993, Knill *et al*. 2002, Castagnino and Lombardi 2004, Castagnino and Fortin 2011a, 2012).

3.- Conceptual challenges of the open-system approach

The theory of decoherence in its EID version has became the "new orthodoxy" in the quantum physicists community (Leggett 1987, Bub 1997). In fact, decoherence is studied in many areas of physics, and has gained a great relevance in quantum computation, where the aim is to take advantage of superpositions and, therefore, to avoid classicality.

Despite its great success, from a conceptual viewpoint there are still some challenges that must be faces if one wants to offer a self-consistent view of the classical limit. We will focus on three of them:

1. *The closed-system problem*: EID cannot be applied to closed systems, in particular to the universe as a whole.
2. *The defining-system problem*: EID does not provide a criterion to decide where to place the "cut" between the proper system and the environment.
3. *The problem of the emergence of the classical world*: Under certain conditions, EID cannot define a unique classical system emerging from the quantum domain.

3.1.- The closed-system problem

According to the authors of the EID approach, the aim of the program "*is to describe the consequences of the 'openness' of quantum systems to their environments and to study the emergence of the effective classicality of some of the quantum states and of the associated observables*" (Zurek 1998: 1793). Therefore, the split of the Universe into the degrees of freedom which are of direct interest to the observer –the system– and the remaining degrees of freedom –the environment– is absolutely essential to understand "*the quantum origin of the classical world*" (Paz and Zurek 2002: 77). Zurek claims that the prejudice that seriously delayed the solution of the problem of the emergence of classicality is itself rooted in the fact that the role of the "openness" of a quantum system was ignored for a very long time (Paz and Zurek 2002, Zurek 2003).

If decoherence explains the emergence of classicality, but only open systems can decohere, the question is: what about closed systems, in particular, the universe as a whole? (Pessoa Jr. 1998). In the literature, several models con be found that describe decoherence in systems with no environment understood in the traditional way. For instance:

- There are systems, such as the Casati-Prosen (2005) model, where decoherence is manifested by the vanishing of the interference pattern on a screen located in a closed cavity. Independently of the details of the models, these are cases where it is not possible to consider that the phenomenon is due to the interaction with an external environment (see also Castagnino 2006).

- In the case of the systems studied by the self-induced decoherence approach, the loss of coherence is attributed to a very generic coarse-graining, and not to the intervention of an environment external to the system (see Castagnino and Fortin 2013 and references therein).
- In the systems studied by Gambini and collaborators (Gambini, Porto and Pullin 2007, Gambini and Pullin 2007, 2010), the authors analyze the influence of an extra term in the evolution equation, which comes from quantum gravity considerations. The term responsible for decoherence does not result from the interaction with an environment, but expresses a coarse-graining due to time-uncertainty.
- Some authors describe decoherence in the Heisenberg representation (Polarski and Starobinsky 1996, Kiefer and Polarski 2009) in the Heisenberg representation. In this formalism the loss of coherence, treated by means of the Bogoliubov transformation, is due to the dynamics of the system itself.

Given the peculiar features of the EID approach, these cases are beyond its application scope. In fact, according to Zurek (1994), since a closed system evolves forever deterministically, the issue of its classicality cannot even be posed.

3.2.- The defining-system problem

Since EID does not apply directly to closed systems, in these cases "internal environments" are defined: the closed system is partitioned into some degrees of freedom representing the system of interest, and the remaining degrees of freedom that play the role of the environment. For example, in the cosmological context, long wavelength modes are usually considered the system, and short wavelength modes are conceived as the environment (Lombardo and Mazzitelli 1996). However, this is not the only way of introducing the split into the closed system. In a recent study of the fluctuations generated during the inflationary period of the cosmic evolution, it is supposed that the tensor and scalar fluctuations interact with each other, and tensor fluctuations act as an environment that causes the loss of coherence of the scalar fluctuations, whose classicality is so justified (Franco and Calzetta 2011).

This cosmological case is only an example of the fact that, although the EID approach studies the correlations between system and environment and also between different subsystems (Paz and Roncaglia 2009), there is a conceptual difficulty in the definition of the systems involved in the phenomenon of decoherence: the approach does not supply a general criterion to discriminate between system and environment. In general, the classically behaving degrees of freedom are assumed in advance: the application of the EID formalism does not predict which observables will show a classical behavior but only confirms a previous assumption. This problem is acknowledged by Zurek himself: “*one issue which has been often taken for granted is looming big, as a foundation of the whole decoherence program. It is the question of what are the "systems" which play such a crucial role in all the discussions of the emergent classicality. This issue was raised earlier, but the progress to date has been slow at best. Moreover, replacing "systems" with, say, "coarse grainings" does not seem to help at all, we have at least tangible evidence of the objectivity of the existence of systems, while coarse-grainings are completely "in the eye of the observer."*” (Zurek 2000: 338; see also Zurek 1998).

3.3.- The problem of the emergence of the classical world

According to the EID approach, decoherence explains the transition from quantum to classical (Zurek 1991), that is, the emergence of the classical world from the quantum realm. This classical world must be objective as decoherence itself, and not be confined to “the eye of the observer”. However, as indicated above, the EID approach provides no criterion to distinguish the system of interest from its environment. As a consequence, in order to apply the EID formalism to a closed system U , we can introduce the decomposition between system and environment in many different ways: $U = S_1 \cup E_1$, $U = S_1 \cup E_1$, …, $U = S_n \cup E_n$. Since there is no privileged decomposition, this situation leads to one of the following three cases:

- If none of the systems S_i arising from the different decompositions decoheres, then classicality does not emerge.
- If none of the systems S_i arising from the different decompositions decoheres except one, say, S_k , then only this subsystem S_k decoheres and becomes classical.
- If more than one of the systems S_i arising from the different decompositions decohere but their union does not, then the classicality emerging from the underlying quantum domain is not univocally determined.

A concrete example of this last case is proposed by Castagnino, Fortin and Lombardi (2010a, see discussion in Lombardi, Fortin and Castagnino 2012): a generalized spin-bath model of $m+n$ spin-1/2 particles, where the m particles interact with each other and the n particles also interact with each other, but the particles of the m group do not interact with those of the n group. The study of the model shows that there are definite conditions under which all the particles decohere, but neither the system composed of the m group nor the system composed of the n group decohere.

This kind of cases poses a conceptual challenge to the EID approach: if classicality is conceived as an objective property, the fact that a system behaves classically or not cannot depend on the way in which the observer decides to split the original closed system. In other words, this situation challenges the spirit of the original EID proposal, according to which decoherence provides the basis of a classic limit that explain the objective emergence of the classical world.

4.- About the reference of the reduced state

The conceptual difficulties derive precisely from its open-system perspective. Therefore, it seems reasonable to reconsider the status of the open systems and, in particular, of what it is supposed to be what represents their behavior: the reduced state.

In classical statistical mechanics, the problem of irreversibility in classical statistical mechanics turns out to be how to account for an irreversible approach to equilibrium in systems ruled by time-reversal invariant laws (see Lombardi 2003, Frigg 2007). The standard answer in the Gibbsian Framework relies on coarse-graining: whereas the statistical state of the system, represented by a density function, evolves according the Liouville theorem, the evolution of coarse-grained states is not constrained by the theorem and, under definite conditions of instability, may approach a definite limit for $t \to \infty$. Of course, there are deep disagreements about the interpretation of the irreversibility so obtained. But, independently of them, nobody ignores the difference between the statistical state, which evolves according to the dynamical postulate of the theory, and the coarse-grained state, which may tend to a final stable state.

The situation in quantum mechanics is quite different: the distinction between the different kinds of states appearing in the quantum discourse is usually not sufficiently emphasized. For instance, it is said that the dynamical postulate of quantum mechanics only applies to closed systems, whereas reduced operators actually represent quantum states of open systems. Nevertheless, we are not informed about the evolution *law* for reduced states; in fact, the evolution of open systems always depends, in the final analysis, on the unitary evolution of the whole closed system of which the open system is a part. It is also admitted that, whereas the states of closed systems embody quantum correlations, reduced states may cancel those correlations and, as a consequence, cannot be used for computations in certain cases. However, in spite of this central difference, the states of closed and open systems are usually treated on equal footing.

Although a minority, some authors have conceived reduced states as coarse-grained states (see, for example, Gell-Mann and Hartle 1993, Omnès 1994, Anastopoulos 2002). Nevertheless, in general the claim does not go beyond pointing out the operation of tracing over the degrees of freedom of the environment. In a previous article we have shown that the reduced state provides a description that can be understood by means of a generalized conception of coarse-graining (Fortin and Lombardi 2014): from being originally conceived as the quantum state of the *open* system, it turns out to be the coarse-grained state of the *closed* system.

In its traditional classical form, coarse-graining is based on the partition of a phase space into discrete and disjoint cells: this mathematical procedure defines a projector Π (see Mackey 1989) that cancels some components of the state vector ρ: only certain components are in the coarse-grained description $\rho_{cg} = \Pi\rho$. If this idea is generalized, coarse-graining can be conceived as a projection that cancels some components of a vector representing a state.

In the classical case, let us recall that the reduced state ρ_1 of S_1 is defined as the density operator by means of which the expectation values of all the observables belonging to S_1 can be computed. Precisely, if $\mathcal{H}_1$ and $\mathcal{H}_2$ are the Hilbert spaces of S_1 and S_2 respectively, $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the Hilbert space of S, $O_1 \in \mathcal{H}_1 \otimes \mathcal{H}_1$ is an observable of S_1, I_2 is the identity in $\mathcal{H}_2 \otimes \mathcal{H}_2$, and $\rho \in \mathcal{H} \otimes \mathcal{H}$ is the state of S, then the reduced state of S_1 is defined as the density operator ρ_{S_1} such that

$$\forall O = O_1 \otimes I_2 \in \mathcal{H} \otimes \mathcal{H}, \quad \langle O \rangle_{\rho} = \langle O_1 \rangle_{\rho_1} \tag{4}$$

Although for dimensional reasons the reduced state ρ_1 cannot be expressed as a direct projection $\Pi\rho$ of the quantum state $\rho \in \mathcal{H} \otimes \mathcal{H}$, the expectation value $\langle O_1 \rangle_{\rho_1}$ can also be expressed as the expectation value of $O = O_1 \otimes I_2$ in a coarse-grained state $\rho_{cg} \in \mathcal{H} \otimes \mathcal{H}$:

$$\langle O_1 \rangle_{\rho_1} = \langle O \rangle_{\rho_{cg}} \tag{5}$$

The density operator ρ_{cg} represents a coarse-grained state because it can be obtained as $\rho_{cg} = \Pi\rho$, where the projector Π executes the following operation:

$$\Pi\rho = \left(Tr_{(2)}\,\rho\right) \otimes \tilde{\delta}_2 = \rho_1 \otimes \tilde{\delta}_2 \tag{6}$$

where $\tilde{\delta}_2 \in \mathcal{H}_2 \otimes \mathcal{H}_2$ is a normalized identity operator with coefficients $\tilde{\delta}_{2\alpha\beta} = \delta_{\alpha\beta} / \sum_{\gamma} \delta_{\gamma\gamma}$.

It is quite clear that ρ_{cg}, although belongs to $\mathcal{H} \otimes \mathcal{H}$, is not the quantum state of S represented by $\mathcal{H}$: it is *a coarse-grained state of the closed system* that disregards certain information of its quantum state. However, ρ_{cg}, supplies the same information about the open system S_1 as the reduced state ρ_1, but now from the viewpoint of the composite system S. Therefore, the reduced density operator ρ_1^r can also be conceived as a kind of coarse-grained state of S that disregards certain degrees of freedom considered as irrelevant.

Once the reduced state is viewed as a coarse-grained state, its non-unitary evolution does not restrict the application of the dynamical postulate nor require a new dynamical postulate: it turns out to be a situation analogous to the familiar case of classical instability, where it is completely natural to obtain irreversible coarse-grained evolution from the underlying reversible dynamics of the unstable system, with no need of restrictions or reformulations of the classical dynamical laws (see Berkovitz, Frigg and Kronz 2006). An author who has emphasized the analogy between the classical statistical case and the quantum case is Omnès (2001, 2002), who has repeatedly claimed that decoherence is a particular case of the phenomenon of irreversibility. Now the claim can be endowed with a more precise meaning: as in the case of classical instability, where the coarse-grained state approaches a final state in spite of the reversible evolution of the statistical state, in environment-induced decoherence the reduced state approaches a diagonal reduced state, in spite of the fact that the quantum state indefinitely follows its unitary evolution.

5.- A closed-system approach to decoherence based on expectation values

If the description given by the reduced state can be recovered from the perspective of the closed system, it is not surprising that, by contrast to the view given by the EID approach, decoherence can be accounted for from a closed-system view. In fact, by following the path opened by Zeh (see Joos et al. 2003), it is possible to study the loss of coherence in a system by considering the internal structure of the expectation values of its observables. In particular, from this perspective decoherence is conceived in terms of the vanishing of the interference terms of the expectation values of certain observables of interest (Castagnino, Laura and Lombardi 2007, Castagnino *et al.* 2008).

On this basis, the approach to decoherence proposed here relies on a strategy consisting in two stances:

- The object of study is always the *closed system*, which is considered from the viewpoint of some relevant observables. Therefore, the state used to describe decoherence is not the reduced state of an open system but a coarse-grained state of the closed system.
- The evolution relevant to decoherence is the evolution of the *expectation values* of the closed system observables.

In the rest of this section, the closed system approach will be developed in the light of these two elements.

5.1.- Expectation values and closed systems

The usual presentations of quantum mechanics place the concept of state in the center of the scene: the description of a system is given by the system's state and its time evolution. However, states in quantum mechanics do not supply the value of the observables of the system, as in the classical case; by contrast, they are the theoretical means for computing the expectation values of all the observables of the quantum system. And such expectations values amount to the information that can be empirically obtained from the system, that is, the information that can be measured. Moreover, given the expectation values of all the observables of a system, it is possible to compute the system's state. Therefore, those expectation values provide a complete description of the system and its time evolution, without resorting to the state. This fact is what supports the expectation-value perspective.

This perspective centered on the observables is in resonance with the algebraic formalism of quantum mechanics (Haag 1993), according to which a quantum system is mathematically characterized by the space $\mathcal{O}$ of the self-adjoint elements of an algebra of operators representing observables, and states are represented by functionals on $\mathcal{O}$. In this theoretical framework, the observables are the basic elements of the theory, and states are secondary elements, defined in terms of the basic ones. Therefore, in order to study the subsystems of a system, it is necessary to consider the spaces of observables corresponding to those subsystems.

Let us consider a closed system U partitioned as $U = S_1 \cup S_2$. If $\mathcal{O}_U$ is the space of observables of U, and $\mathcal{O}_1$ and $\mathcal{O}_2$ are the spaces of observables of S_1 and S_2 respectively, then $\mathcal{O}_U = \mathcal{O}_1 \otimes \mathcal{O}_2$. If ρ_U is the state of U, the reduced states of S_1 and S_2 can be computed by means of partial traces as $\rho_1 = Tr_2 \rho_U$ and $\rho_2 = Tr_1 \rho_U$ respectively. With these three states, the expectation values of all the observables of the corresponding systems can be computed:

$$\langle O_U \rangle_{\rho_U} = Tr\left(\rho_U O_U\right) \qquad \langle O_1 \rangle_{\rho_1} = Tr\left(\rho_1 O_1\right) \qquad \langle O_2 \rangle_{\rho_2} = Tr\left(\rho_2 O_2\right) \tag{7}$$

where $O_U \in \mathcal{O}_U$, $O_1 \in \mathcal{O}_1$ and $O_2 \in \mathcal{O}_2$. But, as it is well known, there are always observables of the composite system U that are not observables of the subsystems. In particular, the expectation values of the observables of the form $O_1 \otimes O_2 \in \mathcal{O}_U$ cannot be computed in terms of the subsystems S_1 and S_2. For this reason, from the perspective centered on the observables, the viewpoint given by the closed system has conceptual priority: any partition into subsystems gives a view that is unavoidably partial, to the extent that it cannot capture the information of all the observables of the composite system.

The closed-system approach, by contrast, allows the computation of the expectation values of the subsystems' observables in terms of the state and the observables of the closed composite system. In the above case, the strategy consists in considering the observables O_{U1} and O_{U2} of U that have the form $O_{U1} = O_1 \otimes I_2 \in \mathcal{O}_U$ and $O_{U2} = O_2 \otimes I_1 \in \mathcal{O}_U$. On this basis, the expectation values of the observables of the subsystems can then be computed as:

$$\langle O_{U1} \rangle_{\rho_U} = Tr\left(\rho_U O_{U1}\right) = Tr\left(\rho_U \left(O_1 \otimes I_2\right)\right) = Tr\left(\rho_1 O_1\right) = \langle O_1 \rangle_{\rho_1} \tag{8}$$

$$\langle O_{U2} \rangle_{\rho_U} = Tr(\rho_U O_{U2}) = Tr(\rho_U (O_2 \otimes I_1)) = Tr(\rho_2 O_2) = \langle O_2 \rangle_{\rho_2} \tag{9}$$

But the closed-system approach is completely general, since not restricted to the study of partitions of the closed system U into subsystems. It allows us to consider any subset $\mathcal{O}_R \subset \mathcal{O}_U$, where the $O_R \in \mathcal{O}_R$ do not need to be of the form $O_1 \otimes I_2$: the O_R are the observables considered relevant in the particular situation of interest.

5.2.- Decoherence in the closed-system approach

On the basis of the above explanation, in the context of the present approach a closed system U will not be split into a system of interest and its environment, but will be partitioned into relevant and irrelevant observables: $\mathcal{O}_U = \mathcal{O}_R \cup \mathcal{O}_I$.

Let us consider the expectation value of a generic relevant observable $O_R \in \mathcal{O}_R$ when the system U is in the state ρ_U:

$$\langle O_R \rangle_{\rho_U} = Tr(\rho_U O_R) = \sum_i (o_{Rii})(\rho_{Uii}) + \sum_{i \neq j} (o_{Rij})(\rho_{Uij}) \tag{10}$$

where the (ρ_{Uii}) and the (o_{Rii}) are the diagonal components, and the (ρ_{Uij}) and the (o_{Rij}) are the non-diagonal components of ρ_U and O_R, respectively, in any basis. The second sum of eq.(10) represents the specifically quantum interference terms of the expectation value. If those terms vanished, the expectation value would adopt the structure of a classical expectation value:

$$\sum_i (o_{Rii})(\rho_{Uii}) \tag{11}$$

where the (o_{Rii}) could be interpreted as possible values and the (ρ_{Uii}) could play the role of probabilities since positive numbers that are less than one and sum to one.

On this basis, from an expectation-value perspective there is decoherence for the relevant observables when the expectation values $\langle O_R \rangle_{\rho_U}$ tend to settle down, in an extremely short time, in a value $k = \sum_i (o_{Rii}) P_i$, where $0 \leq P_i \leq 1$, $\sum_i P_i = 1$:

$$\langle O_R \rangle_{\rho_U(t)} \longrightarrow \sum_i (o_{Rii}) P_i \tag{12}$$

Of course, the P_i are not the diagonal elements of a time-independent state ρ_U, since the state $\rho_U(t)$ of the closed system U always evolves unitarily. Nevertheless, the sum of eq.(12) can also be expressed as

$$\sum_i (o_{Rii}) P_i = \sum_i (o_{Rii})(\rho^d_{Uii}) \tag{13}$$

where the (ρ^d_{Uii}) can be conceived as the components of a kind of coarse-grained state ρ^d_G, diagonal in a certain basis that plays the role of pointer basis. In other words, $\langle O_R \rangle_{\rho_U(t)}$ converges, after an extremely short time, to a value that can be computed as if the system were in a state represented by a diagonal density operator $\rho_G^d(t)$:

$$\langle O_R \rangle_{\rho_U(t)} \longrightarrow \sum_i (o_{Rii}) P_i = \sum_i (o_{Rii}) (\rho_{Uii}^d) = \langle O_R \rangle_{\rho_G^d(t)} \tag{14}$$

Summing up, the decoherence of certain relevant observables of the whole closed system amounts to the fact that the expectation values of those observables tend very rapidly to certain time-independent values that can be computed as their expectation values in a time-independent diagonal state. On this basis, the phenomenon of decoherence can be explained in three general steps (see Castagnino *et al.* 2007, Castagnino *et al.* 2008, Castagnino and Fortin 2013):

- **Step 1:** The space $\mathcal{O}_R$ of relevant observables is defined. Regarding this step, the present proposal agrees with all the other approaches to decoherence, which always select a set of relevant observables in terms of which the time behavior of the system is described: gross observables (van Kampen), macroscopic observables of the apparatus (Daneri), relevant observables (Omnès), observables of the open system (environment-induced decoherence), van Hove observables (self-induced decoherence), etc.

- **Step 2:** The expectation value $\langle O_R \rangle_{\rho_U(t)}$, for any $O_R \in \mathcal{O}_R$, is obtained. This step can be performed in two different but equivalent ways:
  - $\langle O_R \rangle_{\rho_U(t)}$ is directly computed as the expectation value of O_{R} in the unitarily evolving state $\rho_U(t)$.
  - A coarse-grained state $\rho_G(t)$, such that $\langle O_R \rangle_{\rho_U(t)} = \langle O_R \rangle_{\rho_G(t)}$ for any $O_R \in \mathcal{O}_R$, is defined, and its non-unitary evolution (governed by a master equation) is computed.

- **Step 3:** It is proved that $\langle O_R \rangle_{\rho_U(t)} = \langle O_R \rangle_{\rho_G(t)}$ reaches a value $\langle O_R \rangle_{\rho_G^d(t)}$:

$$\langle O_R \rangle_{\rho_U(t)} = \langle O_R \rangle_{\rho_G(t)} \longrightarrow \langle O_R \rangle_{\rho_G^d(t)} \tag{15}$$

  ρ_G^d, diagonal in a certain basis that plays the role of pointer basis

This means that, although the off-diagonal terms of $\rho_U(t)$ never vanish through the unitary evolution, it might be said that the system decoheres *from the observational point of view* given by any relevant observable $O_R \in \mathcal{O}_R$.

6.- Environment-induced decoherence from the closed-system approach

As explained above, the EID approach proves that, in many physical models of systems in interaction with their environments, the non-diagonal terms of the reduced state $\rho_S(t)$ rapidly tend to vanish after an extremely short decoherence time t_D (see eq. (3)): the reduced state approaches a state ρ_S^d that is diagonal in the pointer basis.

In this presentation, Steps 1 to 3 are not explicit. However, EID can be rephrased in such a way that it can be viewed from a closed-system viewpoint.

- **Step 1:** In this case, the whole composite system U, whose space of observables is $\mathcal{O}_U$, is partitioned into a system S, represented by the space of observables $\mathcal{O}_S$, and an environment E, represented by the space of observables $\mathcal{O}_E$. Therefore, the space $\mathcal{O}_R$ of relevant observables of U is $\mathcal{O}_S$, whose members have the form:

$$O_{US} = O_S \otimes I_E \in \mathcal{O}_R \subset \mathcal{O}_U \tag{16}$$

  where $O_S \in \mathcal{O}_S$ is a generic observable of the system S and $I_E \in \mathcal{O}_E$ is the identity operator corresponding to the environment E.

- **Step 2:** The expectation value of any $O_R \in \mathcal{O}_R$ is the expectation value of any $O_{US} \in \mathcal{O}_R$., and can be computed as (see eq. (8)):

$$\langle O_{US} \rangle_{\rho_U} = Tr\left(\rho_U O_{US}\right) = Tr\left(\rho_U \left(O_S \otimes I_E\right)\right) = Tr\left(\rho_S O_S\right) = \langle O_S \rangle_{\rho_S} \tag{17}$$

  This means that $\langle O_{US} \rangle_{\rho_U}$ can also be computed as the expectation value of any observable $O_S \in \mathcal{O}_S$ in the reduced state ρ_S of S.

- **Step 3:** The evolution of the reduced state $\rho_S(t)$ (see eq. (3)) has its counterpart in the evolution of the expectation values. Therefore,

$$\langle O_{US} \rangle_{\rho_U(t)} = \langle O_S \rangle_{\rho_S(t)} \longrightarrow \langle O_S \rangle_{\rho_S^d(t)} = \langle O_{US} \rangle_{\rho_G^d(t)} \tag{18}$$

  In other words, this step is equivalent to the diagonalization of the reduced state.

Summing up, all the results obtained by the EID approach can also be obtained by the closed-system approach. However, the closed-system approach is more general, since makes possible to consider decoherence of completely generic sets of observables, that is, sets of observables considered relevant but that do not define systems. In particular, it can be applied to describe the phenomenon of decoherence in cases where the EID approach could not even be considered.

7.- Solving the conceptual challenges of the open-system approach

As discussed in Section 3, the EID approach faces some challenges which, although not serious in the application of the formalism, undermine the conceptual understanding of the phenomenon of decoherence. These difficulties derive precisely from what was considered the main advantage of the approach: its open-system perspective. For this reason, it is not surprising that they are solved, or dissolved, when the perspective changes and opens the way to a closed-system view.

7.1.- The closed-system problem

Precisely due to its generality, the closed-system approach can be applied to cases that are beyond the scope of the EID approach. For instance, it can be successfully applied to the formalism of self-induced decoherence, which was specifically designed to account for decoherence in closed systems (see Castagnino *et al.* 2007, Castagnino *et al.* 2008). In fact, from this view, if the closed system has continuous spectrum energy, it decoheres in the basis of the energy from the viewpoint of almost all its observables, with the exception of the observables that are not experimentally accessible (see discussion in Castagnino and Lombardi 2004). Precisely due to its closed-system perspective, the self-induced decoherence approach has been successfully applied to closed system models, such as the model of a flat Roberson-Walker universe (Castagnino and Lombardi 2003) and the Casati-Prosen model (Castagnino 2006).

It is interesting to recall that certain presentations of the EID approach suggest a close relationship between decoherence and dissipation: since decoherence is a consequence of the interaction between an open system and its environment, it must always be accompanied by other manifestations of openness, such as the dissipation of energy into the environment. Precisely for this reason, the non-dissipative approaches to decoherence (Polarski and Starobinsky 1996, Bonifacio *et al*. 2000, Ford and O'Connell 2001, Frasca 2003, Sicardi *et al*. 2003, Kiefer and Polarski 2009) were proposed as alternative or rival to the orthodox EID approach. Maximilian Schlosshauer (2007) has clearly stressed that energy dissipation is not a condition for decoherence: loss of energy from the system is a classical effect, leading to thermal equilibrium in the relaxation time, whereas decoherence is a pure quantum effect that takes place in the decoherence time, many orders of magnitude shorter than the relaxation time. From the closed-system approach, since decoherence is not due to the interaction of a system with its environment (Castagnino, Fortin and Lombardi 2010b), the possibility of confusing decoherence and dissipation vanishes.

7.2.- The defining-system problem

The closed-system approach implies, by its own nature, the dissolution of the "looming big" defining-system problem, that is, the problem that there is no criterion to distinguish between the system and the environment. In fact, in that approach, the splitting of the closed system into an open subsystem and an environment is just a way of selecting the relevant observables of the closed system. Since there are many different sets of relevant observables depending on the observational viewpoint adopted, the same closed system can be decomposed in many different ways: each decomposition represents a decision about which degrees of freedom are relevant and which can be disregarded in any case. But there is no privileged or "essential" decomposition; therefore, there is no need of an unequivocal criterion for deciding where to place the cut between "the" system and "the" environment. If all the ways of selecting the relevant observables of the closed system are equally legitimate, decoherence is a phenomenon *relative* to which observables of the whole closed system are considered relevant and which are disregarded in each case (Castagnino, Fortin and Lombardi 2010a, Lombardi, Fortin and Castagnino 2012, see also Lychkovskiy 2013).

These considerations casts new light on the relationship between decoherence and energy dissipation. To the extent that decoherence is a relative phenomenon, no flow of a non-relative quantity from the open system to the environment can account for decoherence. In particular, although energy dissipation and decoherence are in general easily distinguished because of their different timescales, the very reason for their difference is that energy dissipation is not a relative phenomenon but results from the effective flow of a physical entity, whereas decoherence is relative to the observational partition of the whole closed system selected in each situation.

It is worth insisting on the difference between the open-system and the closed-system approaches by emphasizing the difference between the concepts of subjective and relative. The open-system approach conceives open systems as individuals with an objective existence and, on this basis, searches for the open systems that decohere. For this reason, the cut between system and environment is essential. However, since there is no univocal criterion to decide where to place the cut, the decision rests with the observer, that is, turns out to be a subjective matter. From the closed-system perspective, by contrast, the discrimination between relevant and irrelevant observables does not express an objective intrinsic property of a system. The only system objectively and univocally defined is the closed system; the selection of a set of relevant observables is a selection of a kind of observational reference frame in relation to which decoherence is evaluated: for certain sets of relevant observables the interference terms of the expectation values vanish and not for others. This flexibility is what endows the closed-system approach with the capability of studying the behavior of any set of observables, with no need of conceiving one or some of them as the privileged ones.

7.3.- The problem of the emergence of the classical world

As remarked above, if classicality is conceived as an objective property, the fact that a system behaves classically or not cannot depend on the way in which the observer decides to split the original closed system into a system of interest and its environment. The conceptual difficulty is even more serious due to an already pointed out result: in certain situations the fact that classicality emerges in an open system or not depends on in what composite system that open subsystem is embedded. More precisely, given two partitions of a closed system U, $U = S_1 \cup E_1$ and $U = S_2 \cup E_2$, it may be the case that S_1 and S_2 decohere and behave classically, but $S_1 \cup S_2$ does not decohere and, so, it must be admitted that classicality does not emerge in that composite system.

Although admitting that the relative nature of decoherence dissolves the defining-system problem, it does not affect the problem of the emergence of the classical world: even if emergent, classicality is not conceived as a relative property; a given system behaves as classical or not. Nevertheless, the closed-system approach solves the problem due to its focus on observables. Given the closed system U, saying that it decoheres from the perspective of the relevant observables $O_R \in \mathcal{O}_R$ amounts to saying that the interference terms of the expectation values of all the observables belonging to $\mathcal{O}_R$ vanish with the (unitary) time-evolution of U, and this is not a relative fact. In other terms: the observables of U could be considered one by one to see whether they decohere or not, and then the set $\mathcal{O}_{cl}$ of all the classically behaving observables of U can be defined, with no ambiguity of relativity.

When the explanation of the emergence of the classical world in the closed-system approach is understood, it is clear that, strictly speaking, classicality is not a property of systems: thinking in systems that become classical leads to the already mentioned difficulties. The difficulties can be overcome once it is recognized that *classicality is a property of observables*. The emergent classical world is the world described by the observables that behave classically with respect to their expectation values.

8.- A top-down view of quantum mechanics

The advent of the EID approach was received with great enthusiasm in the scientific community. Many authors considered that decoherence supplies the right answer to the measurement problem and the classical limit of quantum mechanics. For instance, under the assumption that the only legitimate demand for a physical theory is the explanation of our perceptions (the "appearances"), d'Espagnat (2000, p. 136) says that "*for macroscopic systems, the appearances are those of a classical world* […] *decoherence explains the just mentioned appearances, and this is a most* *important result*.” In his book on foundations of quantum mechanics, Auletta (2000, p. 289) makes a stronger claim: “*decoherence is able to solve practically all the problems of measurement*.” From a similar perspective, Anderson (2001, p. 492) asserts that “*the word «decoherence»* […] *describes the process that used to be called «collapse of the wave function»*.”

By contrast, many authors, mainly coming from the philosophy of physics, advanced serious warnings about the capability of decoherence for solving those interpretive problems (see, for instance, Healey 1995, Bub 1997, Bacciagaluppi 2008). In particular, it has been stressed that the diagonalization of the reduced state of the system of interest does not imply that the whole composite system acquires a definite property: “*I do not believe that either detailed theoretical calculations or recent experimental results show that decoherence has resolved the difficulties associated with quantum measurement theory*” (Adler 2003, p. 135). Nevertheless, the idea of the power of decoherence for supplying the final account of the classical limit is still in the air in the physics community; for instance, in a very recent article, Crull claims that decoherence is able to tackle many conceptual problems of quantum physics by itself, with no need of interpretation (Crull 2015; see criticism in Vasallo and Esfeld 2015). Therefore, it has been considered that the virtue of decoherence is only to identify the preferred basis that defines the observables of classical-like behavior. For example, Schlosshauer thinks that “*it is reasonable to anticipate that decoherence embedded in some additional interpretive structure could lead to a complete and consistent derivation of the classical world from quantum-mechanical principles*.” (Schlosshauer 2004, p. 1287). In a similar vein, Elby (1994, p. 364) claims that “*decoherence cannot help modal, relative state, or many-world interpretations fend off general metaphysical criticisms. The value of decoherence lies in its ability to pick out a special basis*.” In fact, the theory of decoherence has been frequently used in the many-world interpretation to solve the problem of the preferred basis (Butterfield 2002, Wallace 2002, 2003), considered the main difficulty of Everett’s proposal (see Stapp 2002). Decoherence has also been integrated into the framework of modal interpretations (see Dieks 1989, Lombardi and Dieks 2013), and Bacciagaluppi and Hemmo (1996) have suggested that the definition of the preferred basis given by decoherence would allow modal interpretations to overcome the criticisms of Albert and Loewer (1990, 1993).

The main criticism to the EID approach regarding the solution of the measurement problem is that, even in the case that the open system decoheres in a given basis, the whole closed system is still in a superposition and, therefore, the observables defined by that basis cannot behave classically. This correct criticism is disregarded by those who insist in endowing decoherence with the capability of solving interpretive problems: they continue to conceive open systems as quantum systems of the same kind as closed systems, and their reduced states as legitimate quantum states.

Therefore, when they select certain relevant degrees of freedom to build the reduced state, they think that they are identifying an objectively defined open system, with its objective properties. This open-system approach is *a bottom-up view*, which leads to beginning by the open systems, and to considering their interaction only after that.

In the previous sections we have shown that this approach involves different difficulties, which ultimately derive from ignoring that reduced states are a kind of coarse-grained states defined as the result of particular needs. This means that decoherence is relative to the degrees of freedom selected in each case. Then, if it is supposed that decoherence identifies the systems that behave classically, it is not even clear that the privileged basis is univocally picked up by decoherence: there are different factorizations that may lead to classicality in non-consistent ways.

The closed-system approach, by contrast, is *a top-down view* that begins by studying the whole closed system. Instead of resorting to reduced states, it focuses on the information of interest by selecting the relevant observables of the closed system. This strategy is more general in the sense that it can be applied to cases not covered by the open-system approach. By denying that decoherence applies to open systems, the closed-system approach dissolves the difficulties of the EID approach. In particular, since what decoheres or not are the observables of the closed system, there is a univocal way to define the set of classically behaving observables; therefore, the emergence of classicality, although manifested through expectation values, is a well-defined objective phenomenon.

This top-down view of decoherence and the classical limit fits into the general framework of a top-down view of quantum mechanics, according to which the only legitimate quantum systems are the unitary evolving closed systems. For instance, from this general view, entanglement is also relative to the partition of the closed system into parts that are not characterized as subsystems (Barnum *et al*. 2003, 2004, Viola and H. Barnum 2010). More precisely, from a generalized perspective, entanglement is not a relationship between systems or states, but between sets of observables (Harshman and Ranade 2011). Therefore, the concept of quantum correlations can also be generalized is such a way that it is also relativized to the subalgebras of the algebra of observables of the closed system (Bellomo *et al.* 2014). This top-down view dissolves the so-called “puzzles” about quantum entanglement (Earman 2014), derived from the lack of a univocal criterion to introduce a decomposition into the closed system.

In the interpretive framework, the modal-Hamiltonian interpretation (Lombardi and Castagnino 2008, Ardenghi, Castagnino and Lombardi 2009, Lombardi, Castagnino and Ardenghi 2010) also takes a top-down closed-system perspective. Given the closed system $S = S_1 \cup S_2$, in the case that there is no interaction between S_1 and S_2 and their time-evolutions are governed by the Schrödinger equation, there is no obstacle to consider them legitimate quantum systems, in particular, subsystems of the composite system S. However, if S_1 and S_2 do not follow unitary evolutions according to the dynamical law of quantum mechanics, they are not viewed as subsystems of S but as mere "parts" of it. Those parts are not quantum systems because they lack independent identity: they are conceived as the result of conventional partitions of the whole quantum system S. This conception not only agrees with the well-known holism of quantum mechanics, but also leads to a reinterpretation of indistinguishability, according to which it is not a relation between individual particles belonging to a certain domain, but a symmetry internal to a non-individual and indivisible whole (da Costa and Lombardi 2014).

All these works show that the view that endows closed system with ontological priority is gaining ground in the quantum foundations community. The top-down view of decoherence is one of its different manifestations.

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